

3D Mapping Aided GNSS-Base Cooperative Positioning Using Factor Graph Optimization

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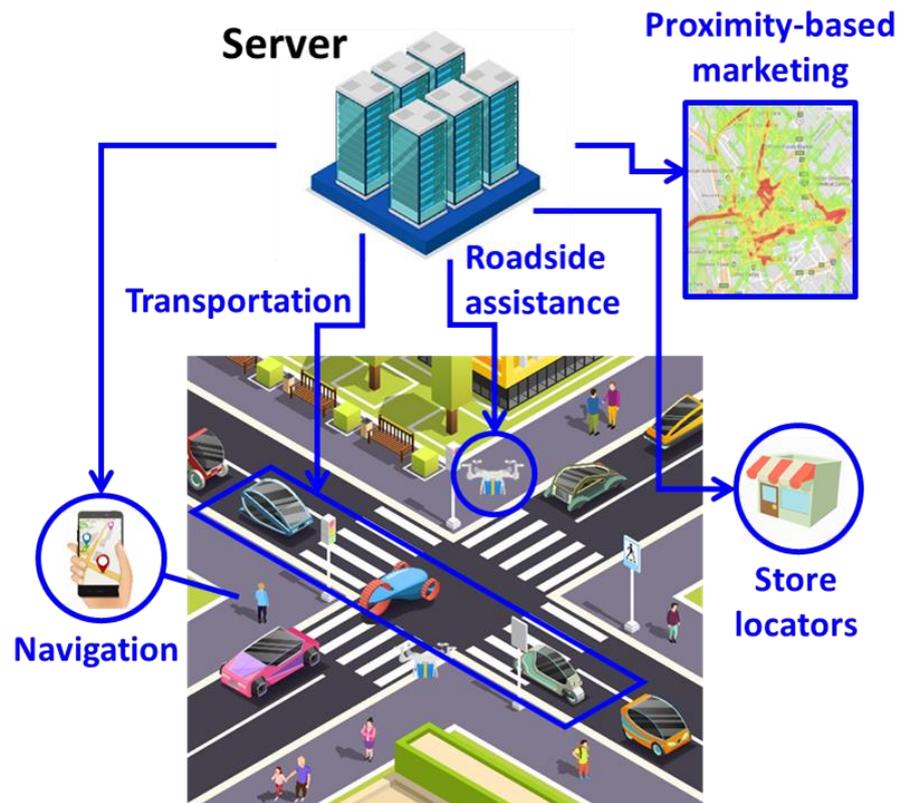
GNSS Data for Location-Based Service (LBS)

The big data can be used to improve the effectiveness of various LBS applications.

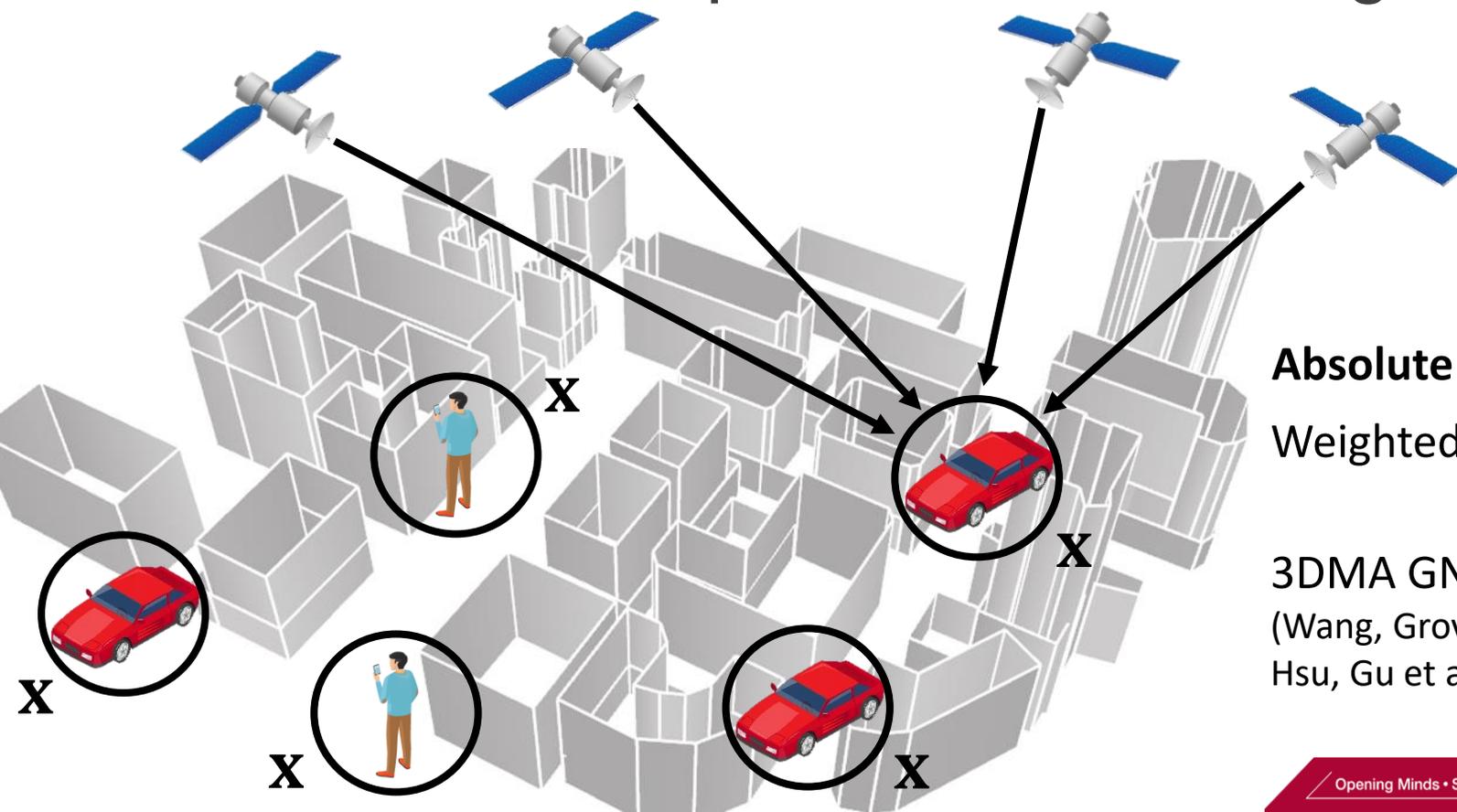
Not accurate in dense urban

Solution:

**3D Mapping Aided (3DMA) GNSS-Based
Cooperative Positioning**



GNSS-Based Cooperative Positioning



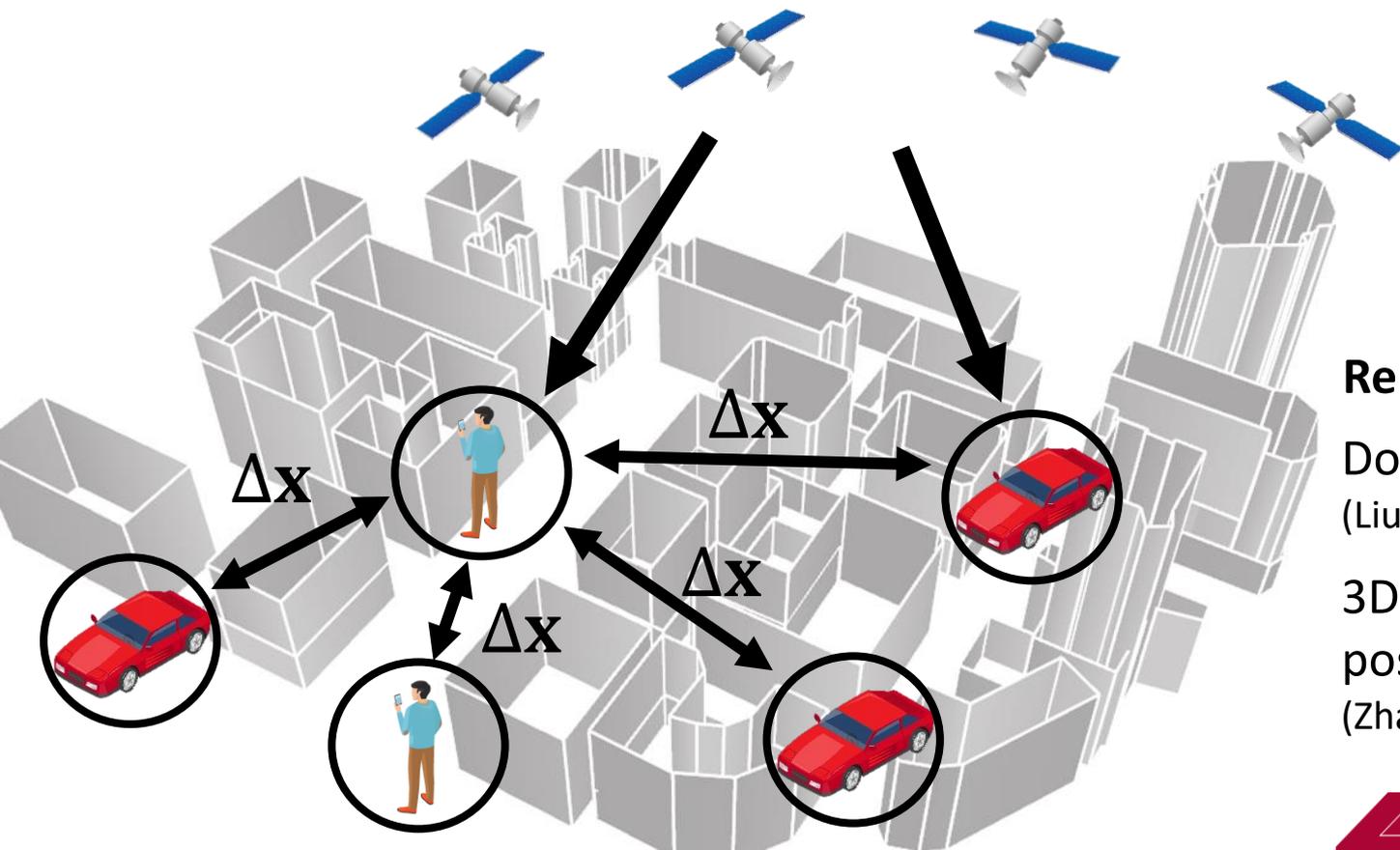
Absolute position

Weighted least squares

3DMA GNSS

(Wang, Groves et al. 2015,
Hsu, Gu et al. 2016)

GNSS-Based Cooperative Positioning



Relative position:

Double difference (DD)
(Liu, Lim et al. 2014)

3DMA cooperative
positioning
(Zhang, Wen et al. 2018)

GNSS-Based Relative Positioning

Pseudorange:

$$\tilde{\rho}_a^i = R_a^i + \delta\rho_a + \delta\rho^i + \epsilon_a^i$$

Receiver clock offset

Atmospheric delay

Satellite clock/orbit bias

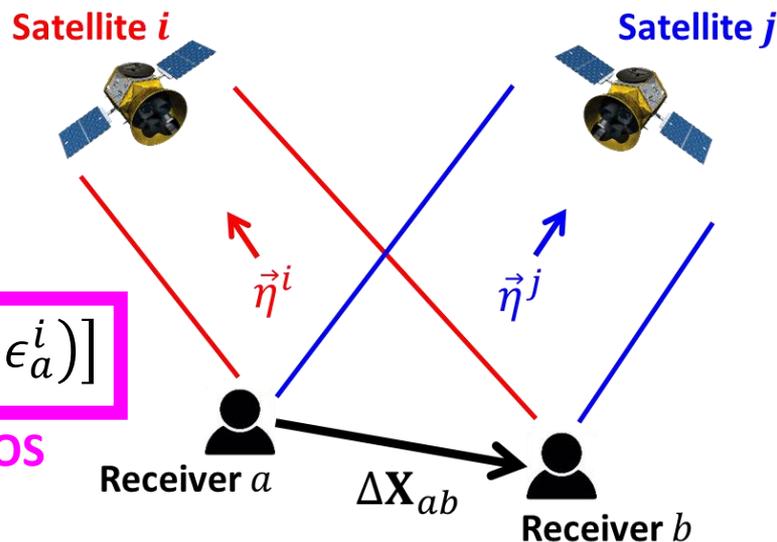
Double difference (DD):

$$D_{ab}^{ij} = (\tilde{\rho}_b^j - \tilde{\rho}_b^i) - (\tilde{\rho}_a^j - \tilde{\rho}_a^i)$$

$$D_{ab}^{ij} = (\vec{\eta}^j - \vec{\eta}^i) \cdot \Delta\mathbf{x}_{ab} + [(\epsilon_b^j - \epsilon_b^i) - (\epsilon_a^j - \epsilon_a^i)]$$

Least square solution:

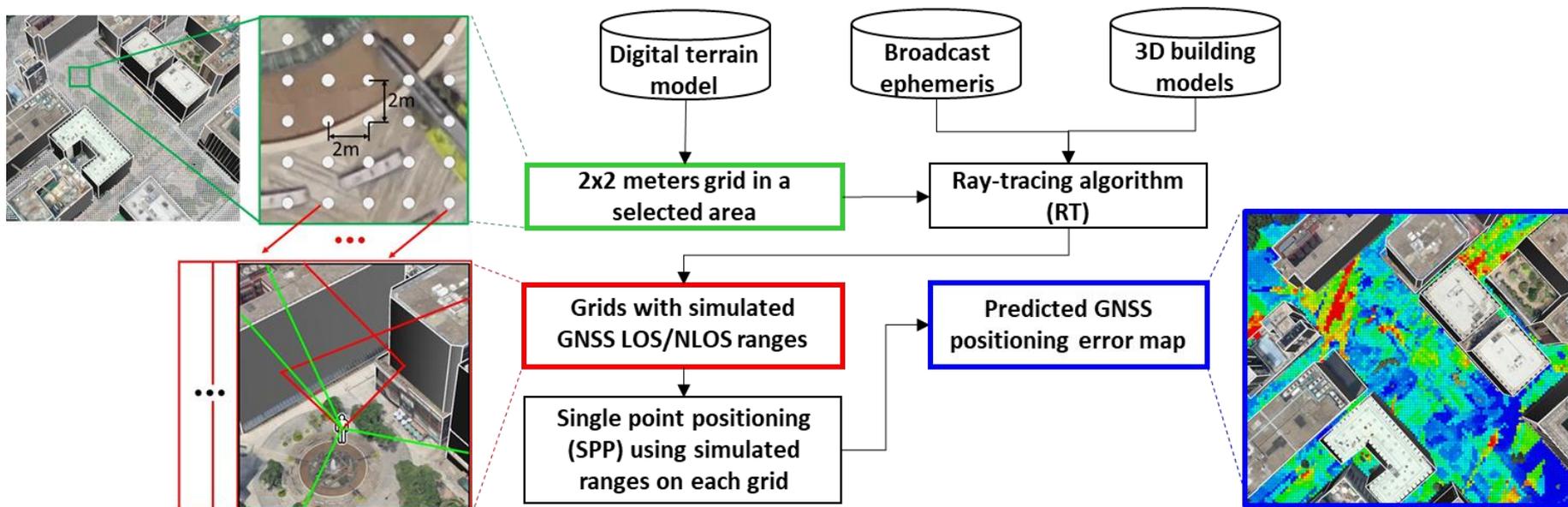
$$\Delta\mathbf{x}_{ab} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{D}_{ab}$$



Multipath and NLOS

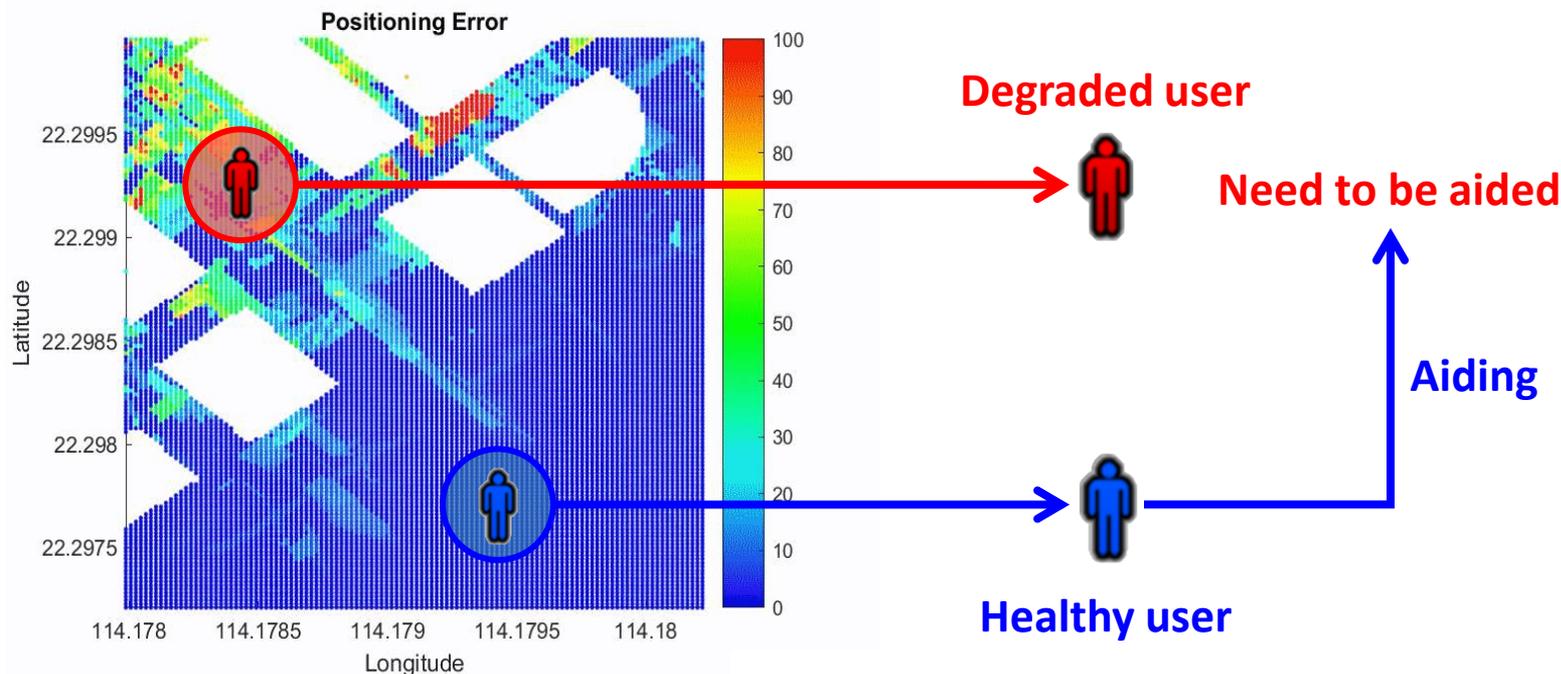
3DMA Cooperative Positioning (ION GNSS+ 2018)

User status evaluation (**healthy**/**NLOS degraded**)



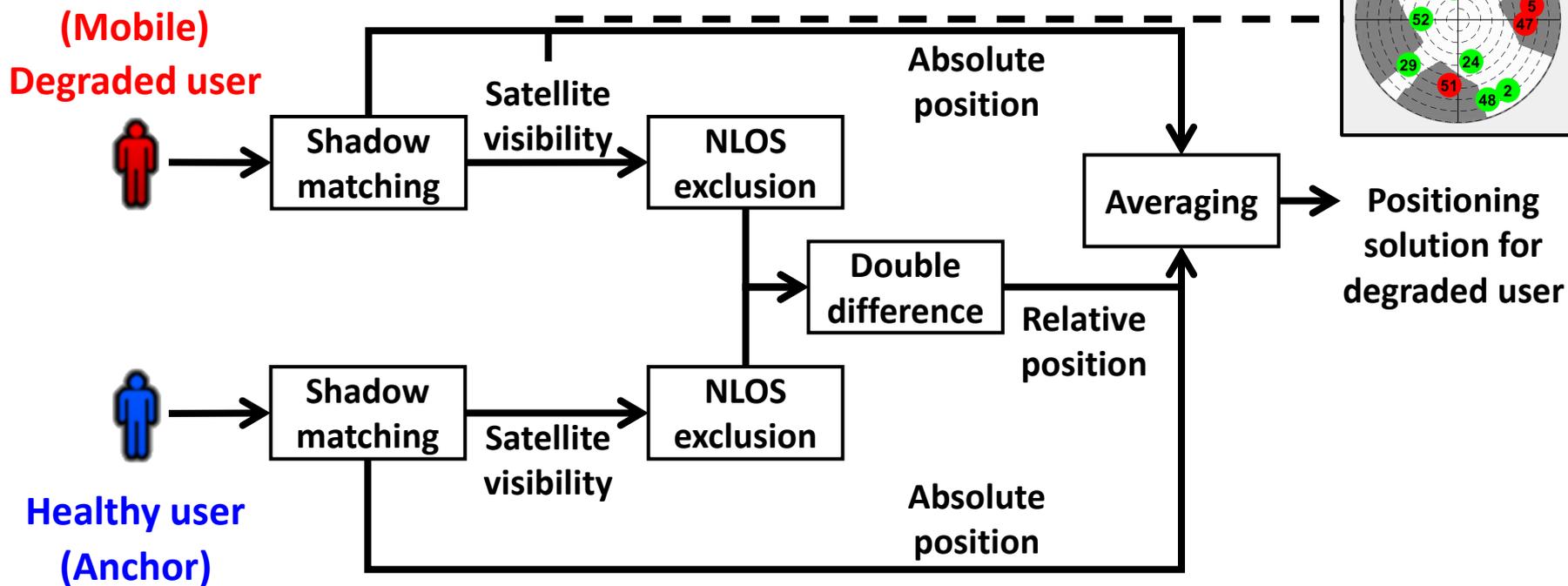
3DMA Cooperative Positioning (ION GNSS+ 2018)

User status evaluation (**healthy/degraded**) for NLOS-mitigation



3DMA Cooperative Positioning (ION GNSS+ 2018)

NLOS mitigation by integrating shadow matching & double difference

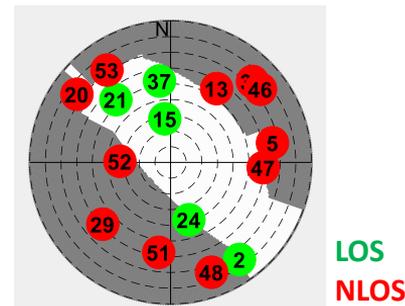


3DMA Cooperative Positioning Performance

Relative positioning error between **Receiver 1** and **Receiver 4**

Method	LS	DD	SDM-DD
RMSE (m)	33.3	84.6	20.3
Availability	100%	100%	70%

LS: Least square positioning
DD: Double difference positioning
SDM-DD: Shadow matching
NLOS-excluded DD

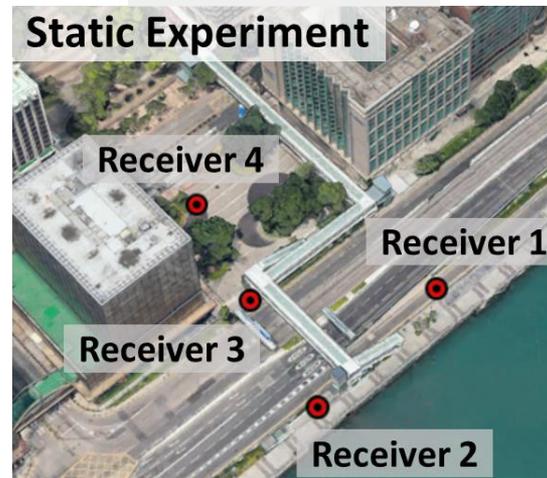


3DMA NLOS-excluded cooperative positioning RMSE (m)

Receiver	1	2	3	4
LS	3.7	5.0	14.7	30.9
SDM-CP	4.2	4.7	14.2	16.2

LS: Least square positioning
SDM-CP: Shadow matching NLOS-excluded cooperative positioning

**Limited measurements
after NLOS exclusion!**



Challenges of GNSS Cooperative Positioning

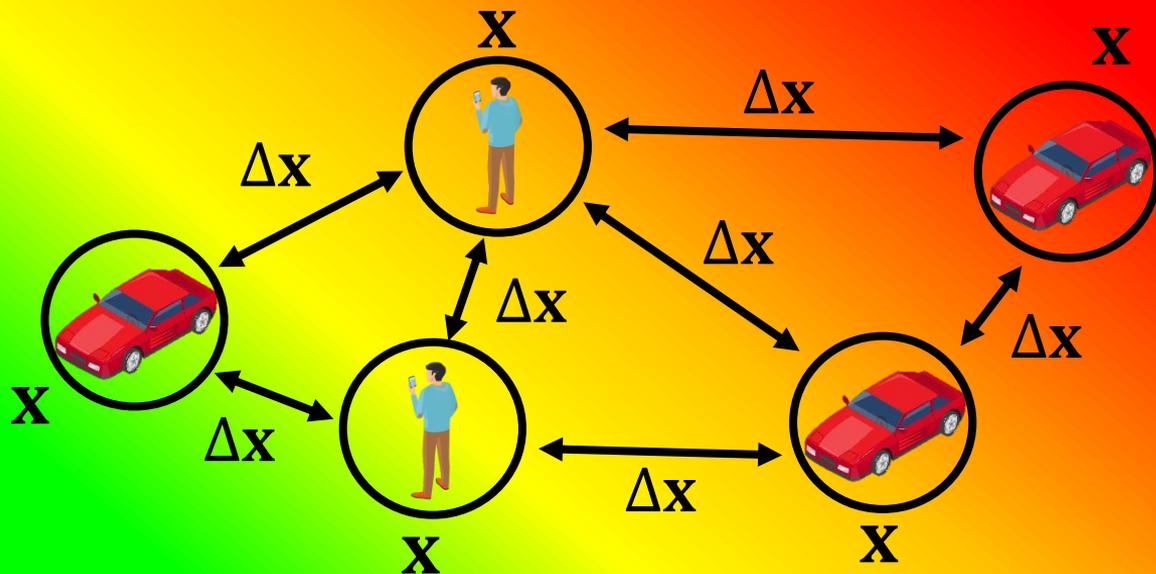
- **Limited** common LOS satellite for cooperative positioning in urban
 - Single anchor-based method is **not robust** for urban
- Proposed solution
- ⇒ Use **ray-tracing algorithm** to **correct NLOS** instead of exclusion
 - ⇒ Use **factor graph optimization** to consider **all** available constraints

Challenges of GNSS Positioning in Urban

Low

Predicted positioning error

High



Environment
information

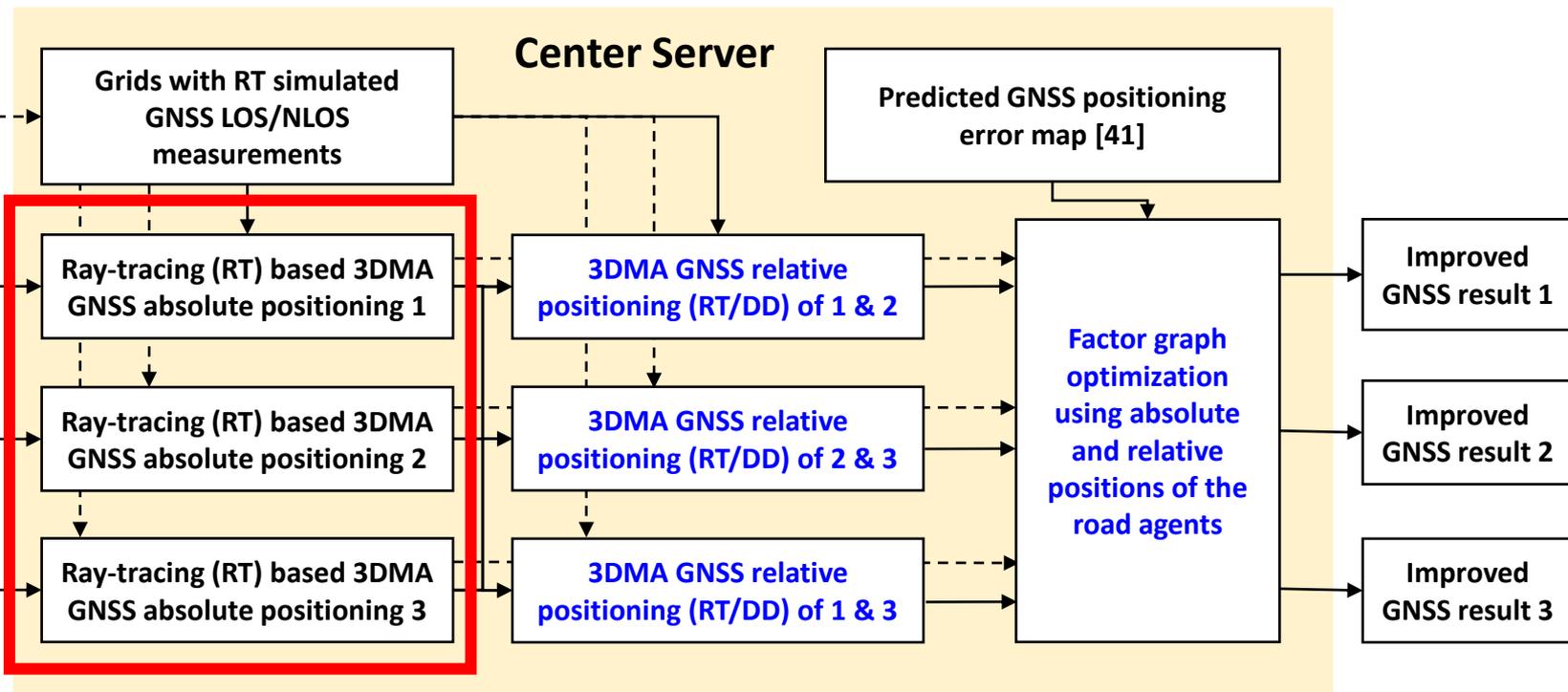


Factor graph
optimization



Each user's position

Flowchart of the proposed algorithm



Ray-tracing algorithm (Hsu, Gu et al. 2016)

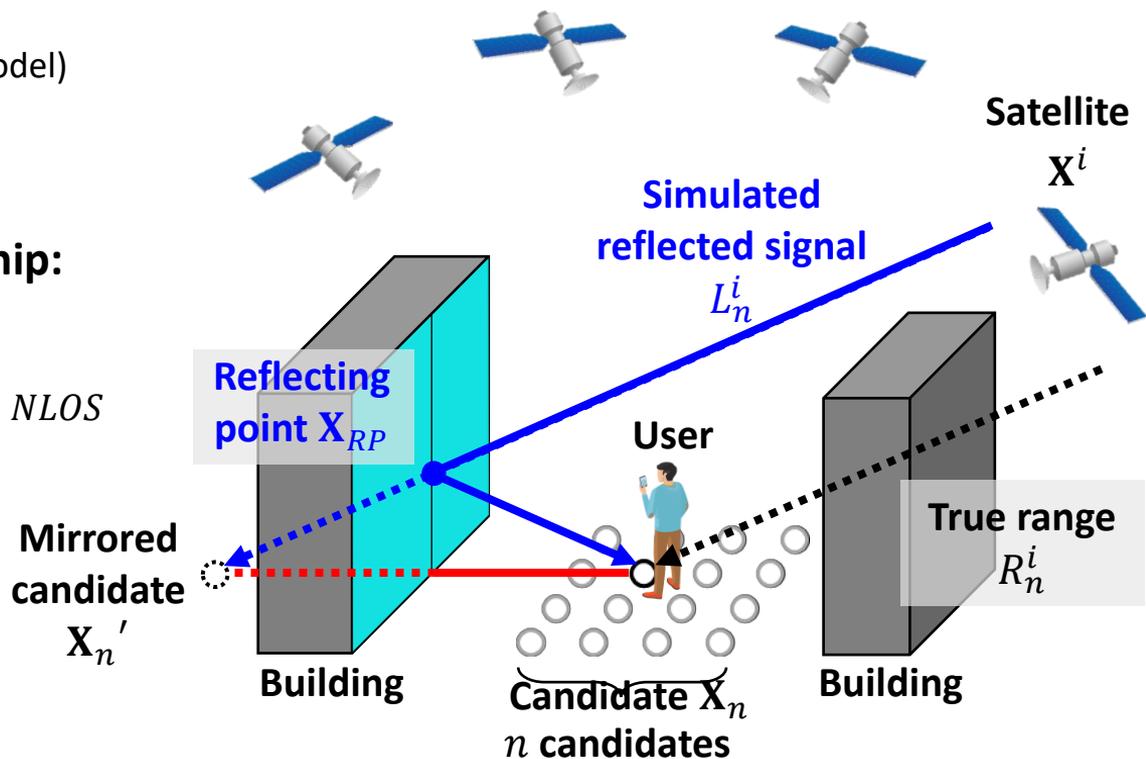
1. Sample candidates (digital terrain model)

2. Simulate GNSS measurements
with NLOS geometrical relationship:

$$L_n^i = \begin{cases} \|\mathbf{X}_n - \mathbf{X}^i\|, & i \in LOS \\ \|\mathbf{X}_{RP} - \mathbf{X}^i\| + \|\mathbf{X}_n - \mathbf{X}_{RP}\|, & i \in NLOS \end{cases}$$

3. GNSS measurement
correction sets:

$$\delta\rho_n^i = L_n^i - R_n^i$$



Ray-tracing algorithm (Hsu, Gu et al. 2016)

4. Simulation-measurement similarity estimation:

(single differenced to cancel receiver clock offset)

$$\text{Simulation: } \hat{S}_n^i = L_n^i - L_n^m$$

$$\text{Measurement: } \tilde{S}^i = \rho^i - \rho^m$$

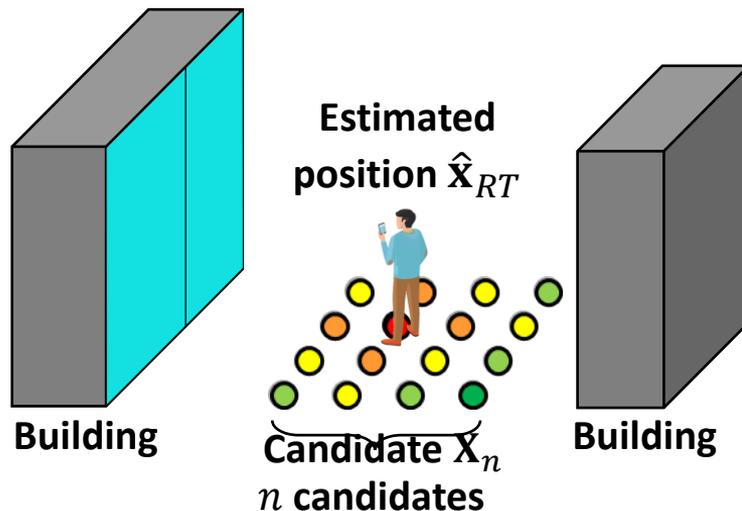
$$\text{Average difference: } \delta S_n = \frac{\sum_i |\hat{S}_n^i - \tilde{S}^i|}{i}$$

$$\Lambda_n = e^{-(\delta S_n - \delta S_{min}) / (\delta S_{max} - \delta S_{min})}$$

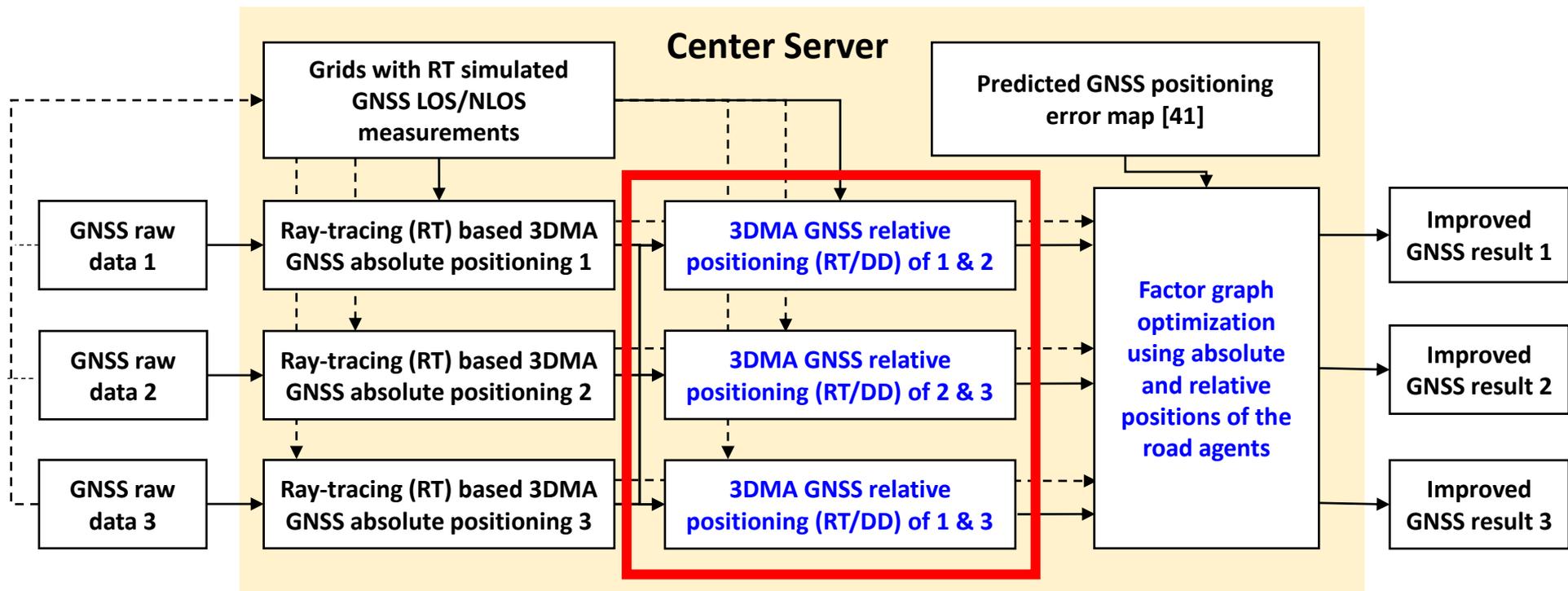
5. Weighted averaging candidate positions:

$$\hat{\mathbf{x}}_{RT} = \frac{\sum_n \Lambda_n \cdot \mathbf{x}_n}{\sum_n \Lambda_n}$$

Low High
Similarity Λ_n



Flowchart of the proposed algorithm

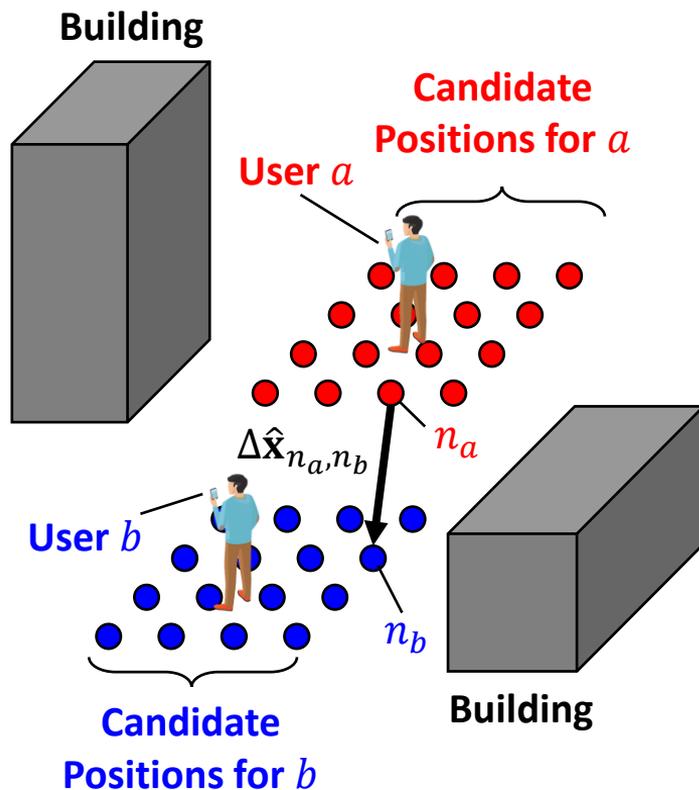


3DMA Double Difference with Ray-tracing

1. Sample candidates for different user
2. Pair each candidates from different user
3. Apply ray-tracing NLOS-corrected double difference relative positioning for each pair:

$$\rho_{n_a}^* = \tilde{\rho}_{n_a} - \delta\rho_{n_a}$$

$$\Delta\hat{\mathbf{x}}_{n_a, n_b} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{D}_{n_a, n_b}^*$$



3DMA Double Difference with Ray-tracing

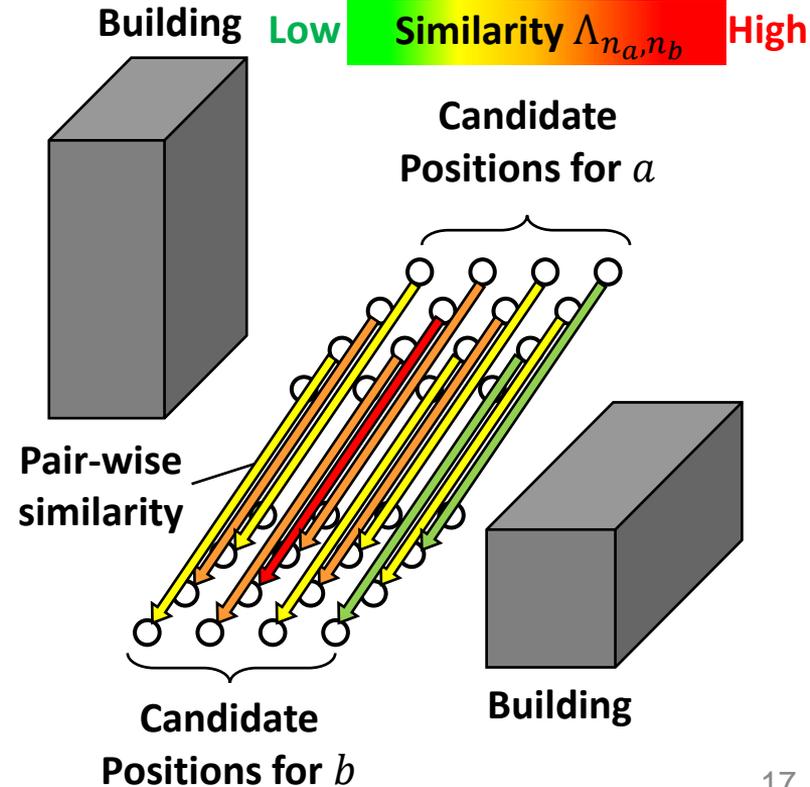
4. Pair-wise simulation-measurement similarity estimation:

$$\delta\Delta\mathbf{x}_{n_a,n_b} = \left\| (\mathbf{x}_{n_b} - \mathbf{x}_{n_a}) - \Delta\hat{\mathbf{x}}_{n_a,n_b} \right\|$$

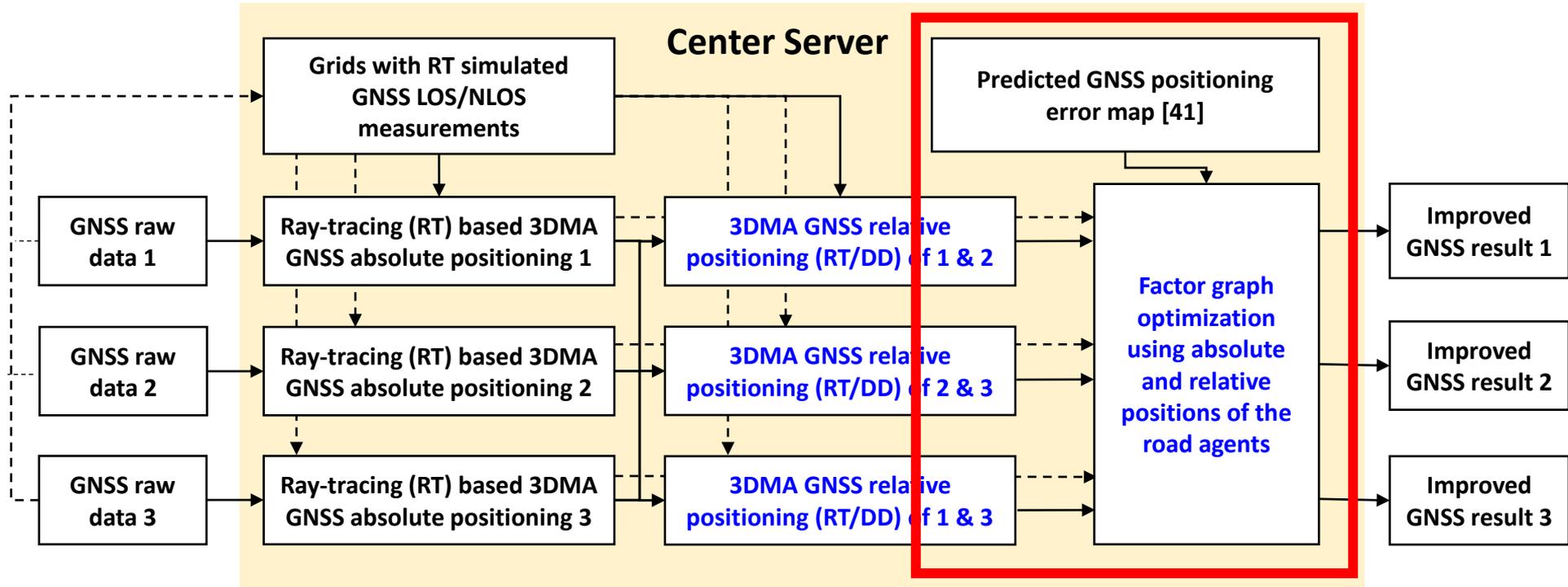
$$\Lambda_{n_a,n_b} = e^{-\left(\delta\Delta\mathbf{x}_{n_a,n_b} - \delta\Delta\mathbf{x}_{min}\right) / \left(\delta\Delta\mathbf{x}_{max} - \delta\Delta\mathbf{x}_{min}\right)}$$

5. Pair-wise similarity-weighted averaging relative position:

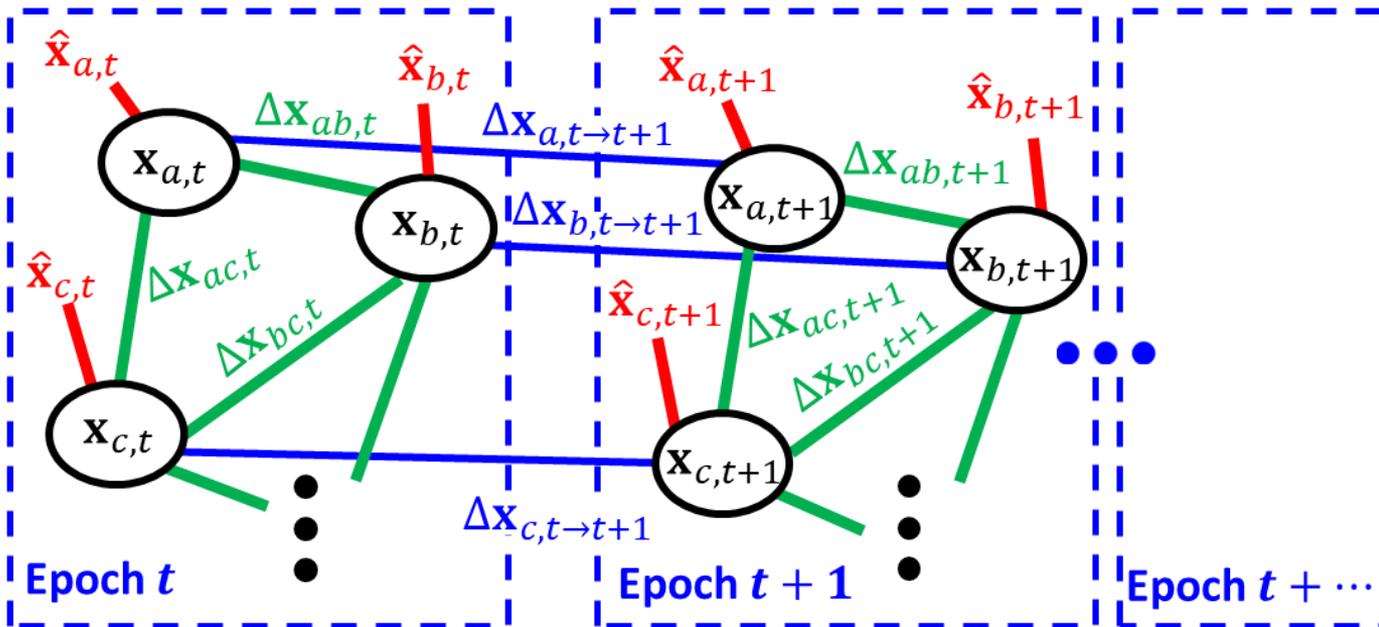
$$\Delta\hat{\mathbf{x}}_{ab} = \frac{\sum_{n_a} \sum_{n_b} \Lambda_{n_a,n_b} \Delta\mathbf{x}_{n_a,n_b}}{\sum_{n_a} \sum_{n_b} \Lambda_{n_a,n_b}}$$



Flowchart of the proposed algorithm



Factor Graph Optimization



-  User's position
-  Absolute position constraint (RT)
-  Relative position constraint (RT-DD)
-  Displacement constraint (Doppler)

Factor Graph Optimization

ε – cost function Ω – covariance matrix

Absolute position constraint:

$$\varepsilon_{a,t} = \mathbf{H}(\mathbf{x}_{a,t}) - \hat{\mathbf{x}}_{a,t} \quad \Omega_{\mathbf{x}_a}^{abs} = \begin{bmatrix} (\overline{\text{err}}_{\mathbf{x}_a}^{east})^2 & 0 \\ 0 & (\overline{\text{err}}_{\mathbf{x}_a}^{north})^2 \end{bmatrix}$$

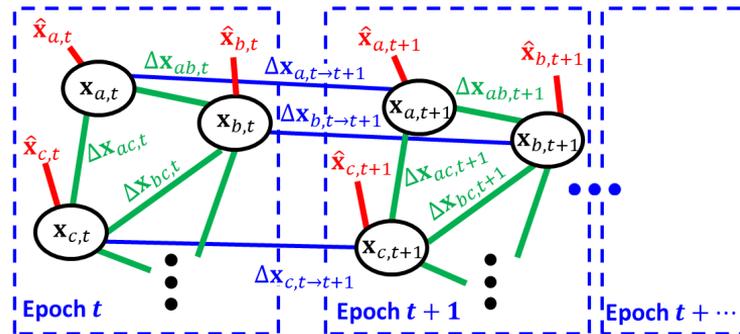
Relative position constraint:

$$\varepsilon_{ab,t} = \mathbf{H}(\mathbf{x}_{b,t}) - \mathbf{H}(\mathbf{x}_{a,t}) - \Delta\hat{\mathbf{x}}_{ab,t} \quad \Omega_{\Delta\hat{\mathbf{x}}_{ab}}^{rel} = \begin{bmatrix} (\overline{\text{err}}_{\mathbf{x}_a}^{east})^2 + (\overline{\text{err}}_{\mathbf{x}_b}^{east})^2 & 0 \\ 0 & (\overline{\text{err}}_{\mathbf{x}_a}^{north})^2 + (\overline{\text{err}}_{\mathbf{x}_b}^{north})^2 \end{bmatrix}$$

Displacement constraint:

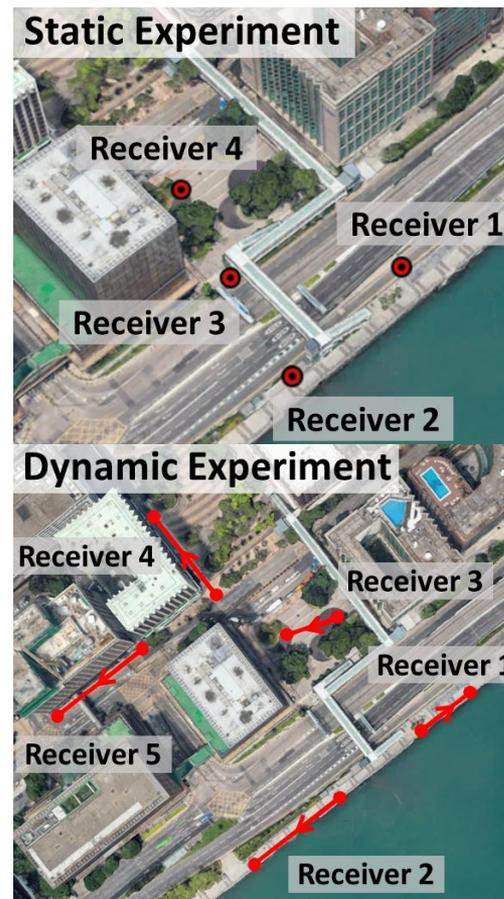
$$\varepsilon_{a,t \rightarrow t+1} = \mathbf{H}(\mathbf{x}_{a,t+1}) - \mathbf{H}(\mathbf{x}_{a,t}) - \Delta\hat{\mathbf{x}}_{a,t \rightarrow t+1} \quad \Omega_{\mathbf{x}_{a,t \rightarrow t+1}}^{displacement} = \begin{bmatrix} \sigma_{a,t \rightarrow t+1} & 0 \\ 0 & \sigma_{a,t \rightarrow t+1} \end{bmatrix}$$

Overall objective function: $\chi^* = \arg \min_{\chi = [\mathbf{x}_1 \ \dots \ \mathbf{x}_N]^T} \sum_k \|\varepsilon_k\|_{\Omega_k}^2$



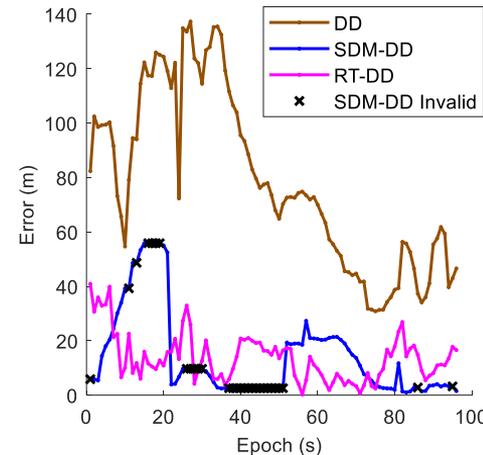
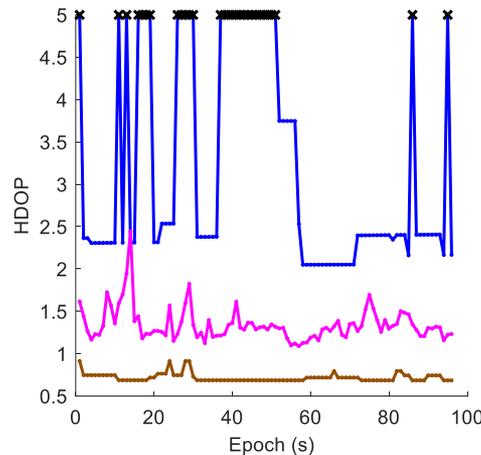
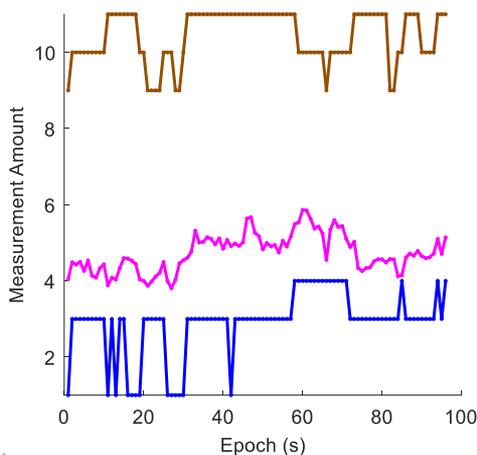
Experiment Setup

Experiment		Static	Dynamic
Receiver type		Ublox EVK-M8T	
Constellation		GPS/GLONASS	GPS/BDS
Environment	Receiver 1	Open sky	Open sky
	Receiver 2	Open sky near bridge	Open sky
	Receiver 3	Building corner under bridge	Between buildings
	Receiver 4	Between buildings	One side building
	Receiver 5		Urban canyon



Experiment Result — Static Experiment

Relative positioning performance between **Receiver 1 (healthy)** and **Receiver 4 (degraded)**

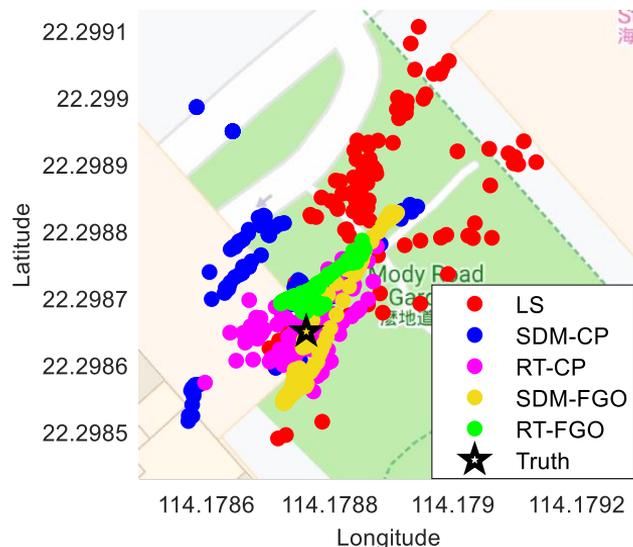
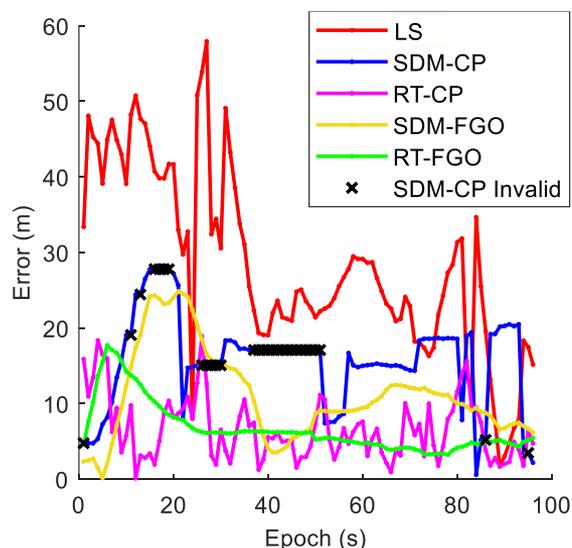


Method	Measurement amount	HDOP	RMSE (m)	Availability
DD	10.4	0.72	84.6	100%
SDM-DD	2.9	2.41	20.3	70%
RT-DD	4.7	1.34	16.3	100%

DD: Double difference positioning
SDM-DD: Shadow matching
NLOS-excluded DD
RT-DD: Ray-tracing NLOS-corrected DD

Experiment Result — Static Experiment

Absolute positioning performance Receiver 4 (degraded)



Method	RMSE (m)
LS	30.9
RT	10.4
SDM-CP	16.2 (70%)
RT-CP	7.8
SDM-FGO	12.6
RT-FGO	7.4

LS: Least squares positioning

RT: Ray-tracing

SDM-FGO: SDM-CP with factor graph

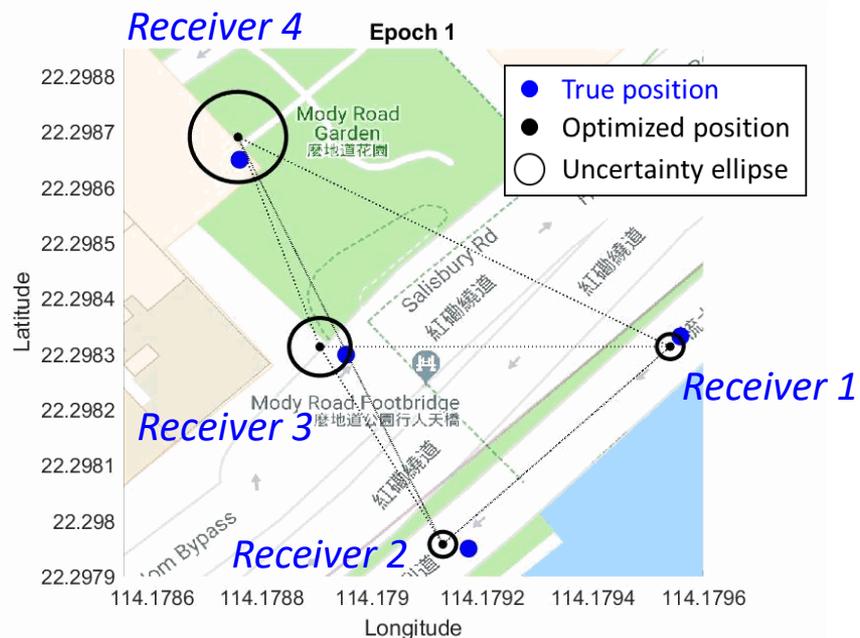
SDM-CP: Shadow matching NLOS-excluded cooperative positioning
(Zhang, Wen et al. 2018 on ION GNSS+ 2018)

RT-FGO: RT-CP with factor graph

RT-CP: Ray-tracing NLOS-corrected
cooperative positioning

Experiment Result — Static Experiment

Absolute positioning performance



Absolute Positioning RMSE (m)

Receiver	1	2	3	4
LS	3.7	5.0	14.7	30.9
RT	2.7	3.1	14.9	10.4
SDM-CP	4.2 (100%)	4.7 (100%)	14.2 (96%)	16.2 (70%)
RT-CP	2.3	3.5	12.5	7.8
SDM-FGO	2.3	2.6	14.7	12.6
RT-FGO	2.7	4.1	8.4	7.4

LS: Least squares positioning

RT: Ray-tracing

SDM-FGO: SDM-CP with factor graph

SDM-CP: Shadow matching NLOS-excluded cooperative positioning

(Zhang, Wen et al. 2018 on ION GNSS+ 2018)

RT-FGO: RT-CP with factor graph

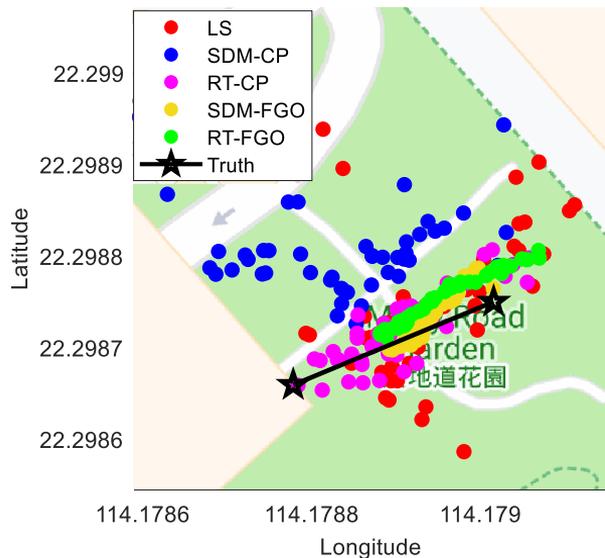
RT-CP: Ray-tracing NLOS-corrected

cooperative positioning

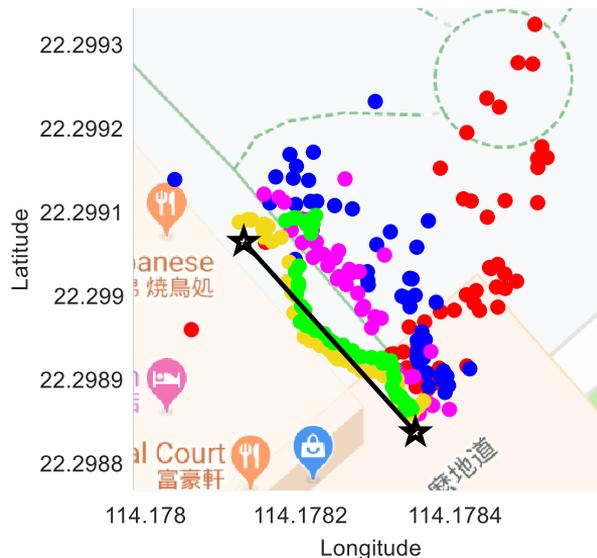
Experiment Result — Dynamic Experiment

Absolute positioning performance

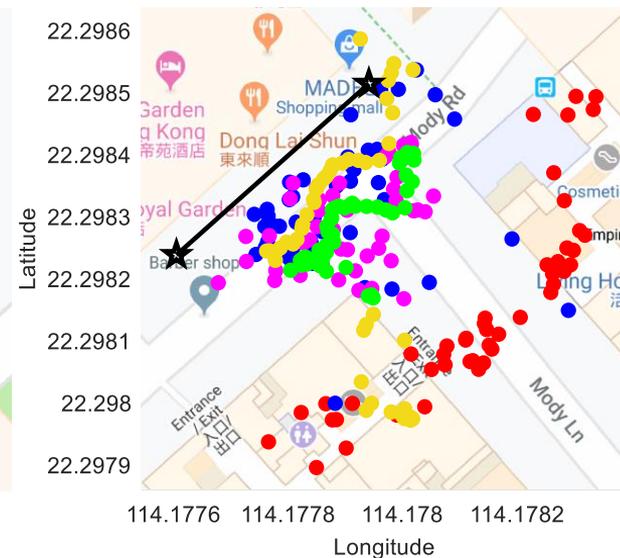
Receiver 3



Receiver 4



Receiver 5



LS: Least squares positioning

SDM-CP: Shadow matching NLOS-excluded cooperative positioning (Zhang, Wen et al. 2018 on ION GNSS+ 2018)

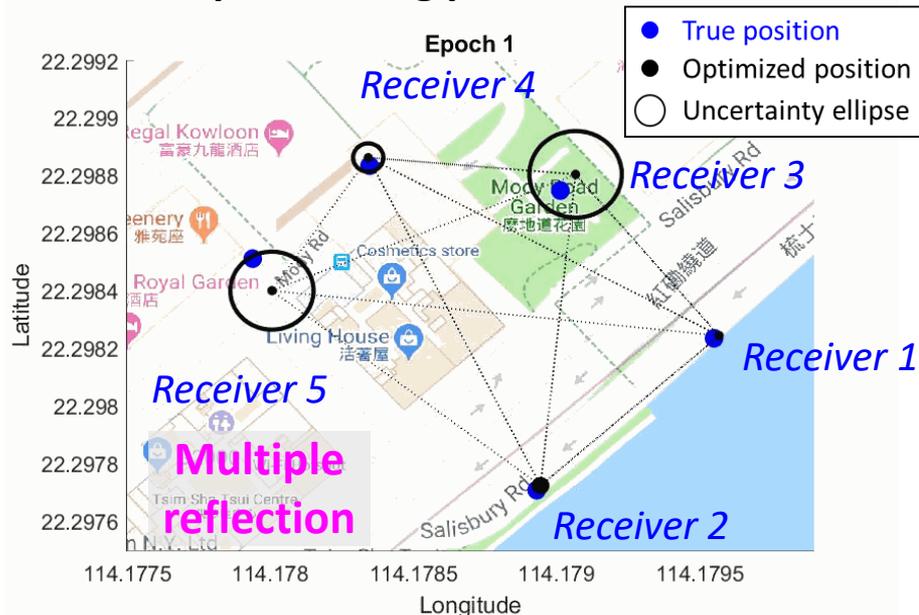
RT-CP: Ray-tracing NLOS-corrected cooperative positioning

SDM-FGO: SDM-CP with factor graph

RT-FGO: RT-CP with factor graph

Experiment Result – Dynamic Experiment

Absolute positioning performance



Absolute Positioning RMSE (m)

Receiver	1	2	3	4	5
LS	4.3	2.0	14.6	25.3	46.5
RT	5.1	3.1	8.5	10.6	20.1
SDM-CP	4.9	2.4	14.7	12.0	18.3
RT-CP	5.4	2.5	5.3	7.6	19.3
SDM-FGO	5.6	1.7	7.5	3.4	25.5
RT-FGO	5.1	2.1	8.1	4.2	18.6

PEM: Positioning error map prediction

LS: Least squares positioning

RT: Ray-tracing

SDM-FGO: SDM-CP with factor graph

SDM-CP: Shadow matching NLOS-excluded cooperative positioning
(Zhang, Wen et al. 2018 on ION GNSS+ 2018)

RT-FGO: RT-CP with factor graph

RT-CP: Ray-tracing NLOS-corrected
cooperative positioning

Conclusion

- The proposed ray-tracing NLOS-corrected DD method improves the relative positioning performance in dense urban.
(factor of 4 comparing to conventional DD with 100% availability)
- The factor graph optimization can improve the robustness by considering all the available constraints. (through space and time)
- The proposed 3DMA cooperative positioning algorithm with factor graph optimization can improve the positioning performance in dense urban.
(25.3m → 4.2m for degraded Receiver 4).

Future Works

- The **scalability** of the proposed algorithm.
- Mitigate **other types of error**.
(Multipath, double reflected NLOS reception, etc)
- The **computation load** for pair-wise candidate matching is large and needs to be reduced.

Q & A

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